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"MUTUAL" ATTRACTION OF A DISLOCATION TO A BIMETALLIC INTERFACE AND A THEOREM ON "PROPORTIONAL" ANISOTROPIC BIMETALS

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Abstract—A straight dislocation parallel to the interface of two perfectly bonded dissimilar linear elastic half-spaces experiences a Peach–Koehler image force tending to move the dislocation toward or away from the interface. As a *rough* rule, the dislocation is repelled from the interface when it resides in the elastically softer of the two half-spaces and is attracted to the interface when it resides in the stiffer half-space. Here we prove that this rough rule holds exactly for "proportional" anisotropic bimetals, i.e. two perfectly joined half-spaces for which $C_{ijkl}\,(2) = \lambda C_{ijkl}\,(1)$, where the constant λ is positive, so that for a dislocation in medium (1) repulsion occurs when $\lambda > 1$ and attraction occurs when $\lambda < 1$. The rough rule does not always hold, since Dundurs and Sendeckyj have shown that for a *pure edge* dislocation in a bimetal composed of two perfectly bonded dissimilar isotropic half-spaces, either (1) the dislocation is attracted to the interface from one side and repelled from the interface from the other side (the usual case), or (2) the dislocation is repelled from the interface no matter which half-space is dislocated. The possibility of attraction from both sides (*mutual attraction*) does not occur for a pure edge dislocation. The present work shows that if the dislocation is *mixed* (if it has both edge and screw components), then mutual attraction in an isotropic bimetal is possible. We delineate how the elastic constants and the screw component must be specified to produce mutual attraction.

1. INTRODUCTION

Dundurs and Sendeckyj (1965) have considered the problem of two joined isotropic linear elastic half-spaces of different elastic constants with one of the half-spaces containing an edge dislocation line parallel to the bimetallic interface. By examining the Peach–Koehler force on the dislocation induced by the presence of a second phase, they were able to show that for all choices of elastic constants corresponding to stable elastic half-spaces either one of two possibilities exists, namely:

(i) the dislocation is attracted toward the interface from one side and is repelled from the interface from the other side; or

(ii) the dislocation is repelled from the interface regardless of which half-space is dislocated.

The possibility of "mutual attraction" to the interface, i.e. attraction of the dislocation to the interface from both sides cannot occur for pure edge dislocations. A brief proof of this result using the Dundurs parameters α and β (Dundurs, 1969) is given in the Appendix. Case (i), which we shall call "attraction–repulsion", is the usual state of affairs; as a rough rule one thinks that attraction toward the interface occurs when the dislocation resides in the elastically "harder" half-space and that repulsion from the interface occurs when the elastically "softer" phase is dislocated. With no precise definition of the terms elastically "harder" and "softer", the rule is only rough and the fact that case (ii) ("mutual repulsion") is a possibility attests to this point. The rough rule is exact for a screw dislocation in a bimetal formed by two perfectly bonded isotropic half-spaces, with the ratio of the respective half-space shear moduli providing the only important measure of "hardness" (Dundurs, 1969).

In the present work we begin with a concise proof that the rough rule mentioned above is *exact* for any dislocation in one of two stable perfectly bonded "proportional" anisotropic linear elastic half-spaces (Lothe, 1992), i.e. two half-spaces whose elastic stiffnesses are related by $C_{ijkl}\,(2) = \lambda C_{ijkl}\,(1)$, where $0 < \lambda < \infty$, so that when $\lambda > 1$ (<1) medium 2 is elastically harder (softer) than medium 1. The remainder of the paper considers an extension of the work of Dundurs and Sendeckyj (1965), namely the possibility that "mutual attraction" may exist for a *mixed* straight dislocation line (both edge and screw character) parallel to the interface between two perfectly bonded dissimilar stable isotropic linear elastic half-spaces. We shall show that there exist both ranges of Burgers vector orientations and choices of elastic constants that permit "mutual attraction". The conditions for mutual attraction can be defined without recourse to numerical computations, although a somewhat lengthy analysis of inequalities is required. We have endeavored to present the analysis in sufficient detail for the interested reader to follow, while at the same time attempting to adhere to space limitations.

2. A THEOREM FOR PROPORTIONAL ANISOTROPIC BIMETALS

Barnett and Lothe (1974) and later Rice (1985) have considered the general problem of a dislocated bimetal consisting of two perfectly bonded anisotropic linear elastic half-spaces and have shown that the image force tending to attract the dislocation toward or repel the dislocation from the interface is given by

$$f^{(i)} = \frac{E(1-2)-E(i)}{h}; \quad i = 1,2. \tag{1}$$

In eqn (1), i refers to the half-space containing the dislocation line (which is straight, infinitely long and parallel to the interface), h is the separation of the dislocation from the interface, $E(i)$ is the pre-logarithmic energy factor for the dislocation when it resides in an infinite medium elastically identical to the half-space denoted by i and $E(1-2)$ is the pre-logarithmic energy factor for the same dislocation when it resides at the interface between the two media. $f > 0$ indicates repulsion from the interface and $f < 0$ indicates attraction. Obviously f vanishes when the two half-spaces are identical.

Now consider that the dislocation resides in medium 1. For our purposes, a more useful form of eqn (1), which may be obtained from the analysis of Ting and Barnett (1993), is

$$2\pi h f^{(1)} = \mathbf{b}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{V}_1^{-1}\mathbf{b}, \tag{2}$$

where $\mathbf{b}$ is the dislocation Burgers vector, the superscript T indicates transposition and the matrices $\mathbf{R}$ and $\mathbf{V}_1$ are given in terms of the impedance matrices $\mathbf{M}_1$ and $\mathbf{M}_2$ for the respective media by (* indicates complex conjugation)

$$\mathbf{R} = \mathbf{M}_1^{-1} + (\mathbf{M}^*)_2^{-1} \tag{3a}$$

$$\mathbf{V}_1 = \mathbf{M}_1^{-1} + (\mathbf{M}^*)_1^{-1}. \tag{3b}$$

$\mathbf{M}_1$, $\mathbf{M}_2$ and their inverses are hermitian and (for stable elastic media) positive definite; hence, $\mathbf{R}$, as well as its inverse, is hermitian and positive definite and $\mathbf{V}_1$, as well as its inverse, is real and positive definite. When the two half-spaces are "proportional", i.e. when $C_{ijkl}\,(2) = \lambda C_{ijkl}\,(1)$, Ting and Barnett (1993) have shown that

$$\mathbf{M}_2^{-1} = \frac{1}{\lambda}\mathbf{M}_1^{-1}. \tag{4}$$

Thus, for a "proportional" anisotropic bimetal with the dislocation in medium 1,

$$2\pi h f^{(1)} = \mathbf{b}^{\mathrm{T}}\mathbf{R}^{-1}(\lambda)\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{R}^{-1}(1)\mathbf{b} \tag{5}$$

with

$$\mathbf{R}(\lambda) = \mathbf{M}_1^{-1} + \frac{1}{\lambda}(\mathbf{M}^*)_1^{-1}. \tag{6}$$

If we compute $\partial f^{(1)}/\partial\lambda$ from eqn (5) and note that, from $\mathbf{R}\mathbf{R}^{-1} = \mathbf{I}$ (the 3×3 identity matrix),

$$\frac{\partial \mathbf{R}^{-1}}{\partial \lambda} = -\mathbf{R}^{-1}\frac{\partial \mathbf{R}}{\partial \lambda}\mathbf{R}^{-1} = \frac{1}{\lambda^2}\mathbf{R}^{-1}(\mathbf{M}^*)_1^{-1}\mathbf{R}^{-1}, \tag{7}$$

we obtain

$$\frac{\partial f^{(1)}}{\partial \lambda} = \frac{1}{2\pi h\lambda^2}\mathbf{b}^{\mathrm{T}}\mathbf{R}^{-1}(\lambda)(\mathbf{M}^*)_1^{-1}\mathbf{R}^{-1}(\lambda)\mathbf{b} > \mathbf{0} \tag{8}$$

since the right side of eqn (8) is a positive definite quadratic (hermitian) form for all nonzero real $\mathbf{b}$. Hence, $f^{(1)}$ is a monotonic increasing function of λ. Since $f^{(1)}$ vanishes when $\lambda = 1$, $f^{(1)}$ is negative for $0 < \lambda < 1$ and is positive for $1 < \lambda < \infty$.

Thus we have proven that "a straight dislocation line parallel to the interface of a 'proportional' anisotropic bimetal is attracted to (repelled from) the interface when the dislocation resides in the elastically harder (softer) half-space". Case (i) mentioned in the Introduction, i.e. "attraction–repulsion", is the only possibility for proportional bonded half-spaces.

3. MUTUAL ATTRACTION FOR MIXED DISLOCATIONS

Consider a bimetal composed of two perfectly bonded isotropic linear elastic half-spaces whose shear moduli and Poisson's ratios are μ_1, ν_1 and μ_2, ν_2, respectively. Without loss of generality we may label the half-spaces so that

$$\Delta\mu = \mu_2 - \mu_1 > 0. \tag{9}$$

The case $\Delta\mu = 0$ does not permit mutual attraction, as can be seen from eqn (15) and the argument preceding inequality (18). It is convenient to define the constants κ_i and τ_i ($i = 1$, 2) by

$$\kappa_i = 3 - 4\nu_i \tag{10a}$$

$$\tau_i = \frac{\mu_i}{1-\nu_i}. \tag{10b}$$

For stable elastic half-spaces, $\mu_i > 0$ and $-1 < \nu_i < \frac{1}{2}$, from which it follows that

$$1 < \kappa_i < 7 \tag{11a}$$

$$0 < \frac{\mu_i}{2} < \tau_i < 2\mu_i. \tag{11b}$$

Let us consider a mixed dislocation of Burgers vector **b** such that $(\pi/2)-\theta$ is the angle between **b** and the dislocation line, i.e.

$$b\,(\text{screw}) = b\sin\theta; \quad b\,(\text{edge}) = b\cos\theta. \tag{12}$$

The pre-logarithmic energy factors in eqn (1) reduce to

$$E(i) = \frac{b^2}{4\pi}[\tau_i\cos^2\theta+\mu_i\sin^2\theta] \tag{13}$$

$$E(1-2) = \frac{b^2}{2\pi}\left[\cos^2\theta\frac{\mu_1\mu_2\{\mu_2(\kappa_1+1)+\mu_1(\kappa_2+1)\}}{\{\mu_2\kappa_1+\mu_1\}\{\mu_1\kappa_2+\mu_2\}}+\sin^2\theta\frac{\mu_2\mu_1}{\mu_1+\mu_2}\right]. \tag{14}$$

Expressing κ_i in terms of τ_i and μ_i allows us to write the force on the dislocation when it resides in the ith phase as

$$\frac{8\pi h}{b^2}f^{(i)} = (A\Delta\mu\pm\Delta\tau)\cos^2\theta\pm\frac{2\mu_i\Delta\mu}{\mu_2+\mu_1}\sin^2\theta \tag{15}$$

where the upper signs go with $i = 1$, the lower signs go with $i = 2$ and

$$\Delta\tau = \tau_2-\tau_1 \tag{16}$$

$$A = \frac{(\tau_1+\tau_2)(\tau_1\tau_2\Delta\mu-4\mu_1\mu_2\Delta\tau)}{(4\mu_1\mu_2-\tau_1\Delta\mu)(4\mu_1\mu_2+\tau_2\Delta\mu)}. \tag{17}$$

By virtue of eqn (9) and inequality (11b), the denominator in eqn (17) is positive so that the sign of A is determined solely by the sign of the second term in the numerator.

If the dislocation under consideration is to experience mutual attraction toward the interface, then both $f^{(1)}$ and $f^{(2)}$ as determined from eqn (15) must be negative. Furthermore, we know that for a pure edge dislocation ($\theta = 0$) the results of Dundurs and Sendeckyj (1965) and Dundurs (1969) show that "it is impossible that both $A\Delta\mu\pm\Delta\tau < 0$". Bearing in mind that we have labeled the half-spaces so that $\Delta\mu > 0$, clearly $f^{(1)}$ is negative if and only if

$$A\Delta\mu+\Delta\tau < 0 \tag{18}$$

and if θ is chosen such that

$$\tan^2\theta < -\frac{A+\dfrac{\Delta\tau}{\Delta\mu}}{2\mu_1/(\mu_1+\mu_2)} = \tan^2\theta_1. \tag{19}$$

If inequality (18) is valid, then the Dundurs and Sendeckyj result for pure edge dislocations requires that

$$A\Delta\mu-\Delta\tau > 0, \tag{20}$$

from which it follows that $f^{(2)}$ will be negative provided that θ is chosen such that

$$\tan^2\theta > \frac{A-\dfrac{\Delta\tau}{\Delta\mu}}{2\mu_2/(\mu_1+\mu_2)} = \tan^2\theta_2. \tag{21}$$

Satisfaction of both inequalities (18) and (20) requires that

$$\frac{\Delta\tau}{\Delta\mu} < A < -\frac{\Delta\tau}{\Delta\mu}, \tag{22}$$

which can only be fulfilled if

$$\Delta\tau < 0. \tag{23}$$

The condition (23) has two important consequences. Firstly, inequality (23) and eqn (9) imply that

$$\frac{1-\nu_1}{1-\nu_2} < \frac{\mu_1}{\mu_2} < 1, \tag{24}$$

so that

$$\nu_2 < \nu_1 \quad \text{and} \quad \kappa_2 > \kappa_1, \tag{25}$$

i.e. if mutual attraction is to occur, the half-space with the smaller shear modulus must have the higher Poisson's ratio. Secondly, inequality (23) ensures that the numerator in eqn (17) is positive so that $A > 0$ and inequality (22) has the sharpened form

$$0 < A < -\frac{\Delta\tau}{\Delta\mu}. \tag{26}$$

In addition to satisfying the inequalities (23) and (26), we must guarantee that $\tan^2\theta_1 > \tan^2\theta_2$ so that there will exist an interval (θ_2, θ_1) within which θ can be chosen. From eqns (19) and (21) this leads to

$$-\frac{A+\dfrac{\Delta\tau}{\Delta\mu}}{2\mu_1/(\mu_1+\mu_2)} > \frac{A-\dfrac{\Delta\tau}{\Delta\mu}}{2\mu_2/(\mu_1+\mu_2)}. \tag{27}$$

which simplifies to

$$A < -\frac{\Delta\tau}{\Delta\mu}\eta, \tag{28}$$

where

$$0 < \eta = \frac{\mu_2-\mu_1}{\mu_2+\mu_1} < 1. \tag{29}$$

When inequality (28) is fulfilled, inequality (26) is automatically fulfilled, so that inequalities (26) and (28) can be replaced by eqn (28) alone.

It remains for us to show that inequality (28) can be fulfilled for choices of the half-space elastic constants which satisfy the stability requirements (according to our labeling) $\mu_2 > \mu_1 > 0$ and $7 > \kappa_2 > \kappa_1 > 1$. To this end let us define

$$J = 4\mu_1\mu_2 - \tau_1\Delta\mu > 0 \tag{30a}$$

$$\omega = \frac{J}{\tau_1\Delta\mu} > 0 \tag{30b}$$

$$s = \frac{\tau_2}{\tau_1}. \tag{30c}$$

Since $0 < \tau_2 < \tau_1$ for mutual attraction to be possible, the allowable range of s to be considered is $0 < s < 1$. Simple algebraic manipulations allow us to reduce the inequality (28) to

$$\frac{(s+1)(1+\omega-\omega s)}{\omega(\omega+s+1)} < (1-s)\eta. \tag{31}$$

At $s = 0$, inequality (31) is satisfied if

$$\eta > \frac{1}{\omega}; \tag{32}$$

as we shall soon prove, inequality (32) is always satisfied for stable elastic half-spaces. At $s = 1$, inequality (31) is satisfied if

$$\frac{2}{\omega(\omega+2)} < 0; \tag{33}$$

since $\omega > 0$, inequality (33) is never satisfied. Thus, if inequality (32) is fulfilled, there will always be some interval $0 < s < \hat{s} < 1$ for which the inequality (31) is satisfied. However, we must also be able to show that the interval $0 < s < \hat{s}$ corresponds to choices of elastic constants which satisfy the stability requirements. Now fulfilment of inequality (32) requires that

$$\frac{\mu_2-\mu_1}{\mu_2+\mu_1} > \frac{\tau_1\Delta\mu}{4\mu_1\mu_2-\tau_1\Delta\mu} \tag{34a}$$

or

$$\frac{1}{\mu_1+\mu_2} > \frac{\tau_1}{4\mu_1\mu_2-\tau_1\Delta\mu} \tag{34b}$$

or

$$\mu_2 < \frac{4\mu_1}{\tau_1}\mu_2 - \mu_2 \tag{34c}$$

or

$$1 < \frac{4\mu_1}{\tau_1} - 1 = 4(1-\nu_1) - 1 \tag{34d}$$

or

$$1 < 3 - 4\nu_1 = \kappa_1, \tag{34e}$$

which is simply one of the stability conditions for medium 1. Hence, stability of medium 1 guarantees that inequality (32) is always satisfied and that inequality (31) is satisfied for all s in $(0, \hat{s} < 1)$.

In order to study the implications of the satisfaction of inequality (31), we note that inequality (31) can be rewritten as

$$G(s) = s^2\omega(1-\eta) - s(1+\eta\omega^2) + (1+\omega)(\eta\omega - 1) > 0. \tag{35}$$

Since $G(0) > 0$ by virtue of satisfaction of inequality (32) and $G(1) = -2$, $G(s)$ has one real zero in (0,1) namely $\hat{s}$; the other zero is real and larger than unity. Introducing the notation

$$\Gamma = \frac{\mu_2}{\mu_1} > 1, \tag{36}$$

so that

$$s = \Gamma\frac{1+\kappa_1}{1+\kappa_2} \tag{37}$$

$$\eta = \frac{\Gamma-1}{\Gamma+1} \tag{38}$$

and

$$\omega = \frac{\Gamma\kappa_1+1}{\Gamma-1}, \tag{39}$$

allows us to deduce that

$$\omega(1-\eta) = \frac{2(\Gamma\kappa_1+1)}{\Gamma^2-1} \tag{40}$$

$$1+\eta\omega^2 = \frac{(1+\kappa_1^2)\Gamma^2+2\Gamma\kappa_1}{\Gamma^2-1} \tag{41}$$

$$1+\eta = \frac{(\kappa_1+1)\Gamma}{\Gamma-1} \tag{42}$$

and

$$\eta\omega - 1 = \frac{(\kappa_1-1)\Gamma}{\Gamma+1}. \tag{43}$$

Using inequality (36) and eqns (37)–(43) shows that $\hat{s}$ is the smallest root of

$$2(\Gamma\kappa_1+1)s^2 - s\Gamma[(1+\kappa_1^2)\Gamma+2\kappa_1] + (\kappa_1^2-1)\Gamma^2 = 0. \tag{44}$$

Thus, the inequality (35) is satisfied if

$$s = \Gamma\frac{1+\kappa_1}{1+\kappa_2} < \frac{1}{4(\Gamma\kappa_1+1)}[\Gamma\{(1+\kappa_1^2)\Gamma+2\kappa_1\} - \Gamma N], \tag{45}$$

where

$$N = [\langle(1+\kappa_1^2)\Gamma+2\kappa_1\rangle^2 - 8(\kappa_1^2-1)(\Gamma\kappa_1+1)]^{1/2}. \tag{46}$$

Canceling Γ from both sides of eqn (45) followed by straightforward algebraic manipulations leads to

$$\kappa_2 > \frac{1}{2(\kappa_1-1)}[(1+\kappa_1^2)\Gamma+2+N]. \tag{47}$$

For mutual attraction, however, we know from inequality (25) that $\kappa_2 > \kappa_1$, which, coupled with the requirement that medium 2 is stable, leads to the extended inequality

$$7 > \kappa_2 > \max\left\{\kappa_1, \frac{1}{2(\kappa_1-1)}[(1+\kappa_1^2)\Gamma+2+N]\right\}. \tag{48}$$

Satisfaction of the inequality (48) (together with the requirements that $\Gamma > 1$ and $1 < \kappa_1 < 7$) allows us to specify ranges of half-space elastic constants and screw dislocation components permitting mutual attraction to occur. The inequality (48) is a particularly convenient form, since the right side of the inequality does not contain κ_2 and the left side is a definite numerical bound. The next section is devoted to an analysis of the implications of this inequality.

4. ANALYSIS OF THE MUTUAL ATTRACTION INEQUALITY

Let us first examine the condition that

$$7 > \frac{1}{2(\kappa_1-1)}[1+\kappa_1^2)\Gamma+2+N], \tag{49}$$

which can be recast as

$$14\kappa_1 - 16 - (1+\kappa_1^2)\Gamma > N. \tag{50}$$

Since N is real [the two roots of eqn (44) are real] and never negative, the left side of inequality (50) must be positive, so that

$$1 < \Gamma < \frac{14\kappa_1 - 16}{1+\kappa_1^2}. \tag{51}$$

When inequality (51) is satisfied, we may square both sides of inequality (50) and rearrange to obtain

$$1 < \Gamma < \frac{25\kappa_1 - 31}{3\kappa_1^2 - \kappa_1 + 4}. \tag{52}$$

The inequality (52) is a stronger condition than inequality (51), i.e. if inequality (52) is satisfied, so is inequality (51). We can select a range of Γ satisfying inequality (52) if and only if the right side of inequality (52) is larger than unity, which leads to

$$(3\kappa_1 - 5)(\kappa_1 - 7) < 0, \tag{53}$$

which, since $1 < \kappa_1 < 7$, requires that

$$\kappa_1 > \tfrac{5}{3} \text{ and thus } \nu_1 < \tfrac{1}{3}. \tag{54}$$

It remains for us to determine which is the larger of the two bracketed terms on the right side of inequality (48). Suppose that

$$\kappa_1 > \frac{1}{2(\kappa_1 - 1)}[(1+\kappa_1^2)\Gamma + 2 + N], \tag{55}$$

which is equivalent to

$$2\kappa_1(\kappa_1 - 1) - (1+\kappa_1^2)\Gamma - 2 > N \tag{56}$$

or

$$2(\kappa_1^2 - \kappa_1 - 1) - (1+\kappa_1^2)\Gamma > N. \tag{57}$$

We shall now prove that inequality (57) is never satisfied for $\Gamma > 1$ and $1 < \kappa_1 < 7$. Since N is nonnegative, the left side of inequality (57) must be positive and, if this is the case, squaring both sides of inequality (57) yields, after rearrangement and cancellations, the inequality

$$(1-\Gamma)(\kappa_1^2 - 1)(\kappa_1 - 1)^2 > 0. \tag{58}$$

With $\Gamma > 1$ and $1 < \kappa_1 < 7$, the inequality (58) is never satisfied. Thus, inequalities (57) and (55) are never satisfied when medium 1 is stable and hence κ_1 is always the smaller of the two bracketed terms on the right side of the inequality (48). As a result, the "mutual attraction" inequality (48) takes the unambiguous form

$$7 > \kappa_2 > \frac{(1+\kappa_1^2)\Gamma + 2 + N}{2(\kappa_1 - 1)}. \tag{59}$$

In addition, the inequalities (54) and (52) must be satisfied.

5. SELECTION OF ELASTIC CONSTANTS PERMITTING MUTUAL ATTRACTION

We may now state a simple algorithm for selecting isotropic half-space elastic constants which admit the possibility of "mutual attraction".

(1) Pick $\mu_1 > 0$.
(2) Pick

$$\tfrac{5}{3} < \kappa_1 < 7 \quad (-1 < \nu_1 < \tfrac{1}{3}).$$

(3) Select μ_2 so that $\Gamma = \mu_2/\mu_1$ satisfies

$$1 < \Gamma < \frac{25\kappa_1 - 31}{3\kappa_1^2 - \kappa_1 + 4}.$$

For the allowable range of κ_1, Γ will lie between 1 and 1.58.
(4) Choose κ_2 so that

$$7 > \kappa_2 > \frac{(1+\kappa_1^2)\Gamma+2+N}{2(\kappa_1-1)},$$

with N given by eqn (46). It is easy to show that κ_2 so chosen will be greater than 3, so that $-1 < \nu_2 < 0$.

The bimetal so constructed is stable and yields "mutual attraction" provided that the screw component of the dislocation is selected so that $\theta_2 < \theta < \theta_1$, where θ_1 and θ_2 are defined by eqns (19) and (21). Our algorithm for choosing appropriate elastic constants for the two half-spaces guarantees that the interval (θ_2, θ_1) exists.

6. SUMMARY AND CONCLUSIONS

We have shown that two perfectly bonded stable "proportional" anisotropic linear elastic half-spaces allow only "attraction–repulsion" to occur. For two perfectly bonded dissimilar stable isotropic half-spaces "mutual attraction" is a possibility for mixed dislocations, provided that the half-space elastic constants and the screw component are suitably chosen. The medium with the lower shear modulus must have the higher Poisson's ratio and that Poisson's ratio must lie between -1 and 1/3 (the smaller Poisson's ratio must be negative), while the ratio of higher to lower shear modulus cannot exceed 1.58. Thus, a stable isotropic half-space whose Poisson's ratio is larger than 1/3 cannot be perfectly bonded to any other stable isotropic half-space so as to permit "mutual attraction". These results have helped Lavagnino and Barnett (1994) to investigate the possibility of mutual attraction occurring in *anisotropic* bicrystals. They have shown that even in identical half-spaces slightly misoriented to produce twist boundaries, either mutual attraction or mutual repulsion is always possible in all crystal classes for all real materials.

We close by acknowledging the debt of gratitude which all who have worked in dislocation elasticity for the past three decades owe to Professor John Dundurs. It is a privilege and a great pleasure to have been extended the opportunity to contribute to this special volume in his honor.

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APPENDIX

In Dundurs [1969, eqns (5.15) and (5.20); the quantity b_x^2 in eqn (5.20) should read b_y^2] it is shown that a pure edge dislocation in medium 1 of an isotropic bimetal is attracted to the interface if

$$\frac{\alpha+\beta^2}{1-\beta^2} < 0, \tag{A1}$$

where α and β are the Dundurs parameters given by

$$\alpha = [\Gamma(\kappa_1+1)-(\kappa_2+1)]/[\Gamma(\kappa_1+1)+\kappa_2+1] \tag{A2a}$$

$$\beta = [\Gamma(\kappa_1-1)-(\kappa_2-1)]/[\Gamma(\kappa_1+1)+\kappa_2+1] \tag{A2b}$$

and

$$\Gamma = \frac{\mu_2}{\mu_1} > 0, \quad 1 < \kappa_i = 3-4\nu_i < 7. \tag{A3}$$

Since interchanging the labeling 1 and 2 is the same as letting $\alpha \to -\alpha$ and $\beta \to -\beta$, the same dislocation in medium 2 is attracted to the interface if

$$\frac{-\alpha+\beta^2}{1-\beta^2} < 0. \tag{A4}$$

But β can be written in the two equivalent forms

$$\beta = 1-\frac{2(\Gamma+\kappa_2)}{\Gamma(\kappa_1+1)+\kappa_2+1} = -1+2\frac{\Gamma\kappa_1+1}{\Gamma(\kappa_1+1)+\kappa_2+1}, \tag{A5}$$

which clearly shows that for stable half-spaces

$$-1 < \beta < 1 \quad \text{so that} \quad 1-\beta^2 > 0. \tag{A6}$$

Thus, mutual attraction, which requires that both eqns (A1) and (A4) be satisfied, also requires that [adding eqns (A1) and (A4)]

$$\frac{2\beta^2}{1-\beta^2} < 0, \tag{A7}$$

which is impossible because of inequality (A6). This proves that mutual attraction for a pure edge dislocation is impossible in a bimetal composed of two perfectly bonded isotropic linear elastic half-spaces. Although Dundurs [1969, the sentence following eqn (4.11)] restricts κ_1 and κ_2 to the range $1 < \kappa_i < 3$, the above proof is valid for the full range $1 < \kappa_i < 7$ (i.e. negative values of Poisson's ratios are allowed for both phases).